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Aharonov–Bohm effect in a mesoscopic ring in a nonstationary magnetic field

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Abstract. Quantum corrections to the conductivity of a mesoscopic ring in a nonstationary magnetic field with the flux $\Phi(t) = Vt$ are investigated. The time dependence of the quantum corrections shows periodic cusps with a period corresponding to the magnetic flux quantum $\Phi_0 = hc/2e$. If the characteristic time of the magnetic flux variation $\tau_0 = \Phi_0/V$ is much smaller than the phase relaxation time τ_φ, the cusps become asymmetrical.

1. Introduction

Periodic oscillations of magnetoconductivity, related to the Aharonov–Bohm effect, are well known. The first prediction of this phenomenon in dirty conductors [1] was experimentally confirmed by magnetoresistance measurements with a hollow, thin-walled metallic cylinder [2] and planar honeycomb network [3]. The effect originates from the interference of electronic paths running around the hole in opposite directions, and has the period of oscillation Φ_0.

The conductivity of disordered mesoscopic rings in stationary magnetic and variable electric fields was studied experimentally in [4]. In particular, the conductivity oscillations with a period h/e were observed in all ranges of electric field frequencies. The oscillations are suppressed if the electric field frequency exceeds the inverse sample transit time and the thermal frequency $kT/\hbar$.

A theoretical study of the stationary Aharonov–Bohm effect in a mesoscopic ring was carried out by Fal'ko [5], with the influence of tunnel contacts taken into account.

The influence of variable magnetic field with the flux $\Phi(t) = Vt + \sum_i A_i \sin(\omega_i t + q_i)$ on the free-electron states was studied in the recent works [6, 7]. The interference of two electron beams is sensitive to V and ω. A stationary interference picture appears at frequencies equal to a multiple of V.

The accumulation of energy of linearly varying (with time) magnetic flux in a quantum ring with limited number of impurities was studied in [8].

However, realization of quantum rings without impurities or with a low amount of impurities is complicated by their small size. In addition, the small ring area leads to a small magnetic flux variation rate, which complicates the observation of the specific effects of a nonstationary magnetic field. Measurements of the Aharonov–Bohm effect in solids are usually carried out using mesoscopic rings [9] whose perimeter is comparable with the phase-breaking length for electrons and much exceeds the mean free path of electrons. The electronic states are strongly mediated by the disorder producing diffusion and cooperon modes.

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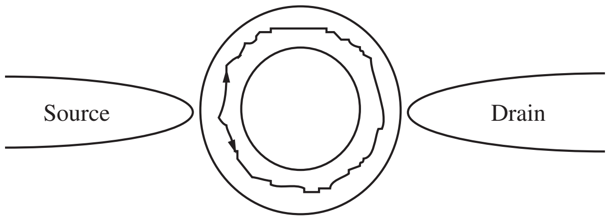

Figure 1. Mesoscopic ring with tunnel contacts. Two interfering trajectories are drawn inside the ring.

In this work, the influence of a linearly varying magnetic field on the quantum corrections to conductance of a disordered mesoscopic ring is studied and the possibility of experimental observation of the effect is analysed. The ring conductance essentially depends on the tunnel contacts (figure 1). We consider tunnelling to be weak, so that contacts have no effect on the states in the ring. The ring width is expected to be sufficiently small, so that the ring can be considered to be a one-dimensional 'wire' from the standpoint of quantum corrections. Therefore, the ring width must be less than $L_\varphi = \sqrt{D\tau_\varphi}$.

2. Calculation of quantum corrections

The analytical expression for the quantum corrections is given in terms of the cooperon Green functions [10] satisfying the diffusion-like equations

$$\left\{\partial_\eta + D\left[-\mathrm{i}\nabla - \frac{e}{c\hbar}\boldsymbol{A}\left(t-\frac{\eta}{2}\right) - \frac{e}{c\hbar}\boldsymbol{A}\left(t+\frac{\eta}{2}\right)\right]^2 + \frac{1}{\tau_\varphi}\right\} C^t_{\eta,\eta'}(x,x') \\ = \delta(x-x')\delta(\eta-\eta') \tag{1}$$

inside the ring.

In a mesoscopic system, equation (1) has to be supplemented with boundary conditions at the contacts. According to [5], they can be introduced phenomenologically in the form of continuity and current conservation equations

$$\begin{aligned} &C(+0,x') = C(L-0,x') \\ &C\left(\frac{L}{2}-0,x'\right) = C\left(\frac{L}{2}+0,x'\right) \\ &2\alpha^{-1}C(0,x') = L\partial_x C(+0,x') - L\partial_x C(L-0,x') \\ &-2\alpha^{-1}C\left(\frac{L}{2},x'\right) = L\partial_x C\left(\frac{L}{2}-0,x'\right) - L\partial_x\left(\frac{L}{2}+0,x'\right). \end{aligned} \tag{2}$$

Here the variable $0 < x < L$ denotes the coordinates along the ring and $L = 2\pi R$ is the ring circumference. According to [5], the parameter $\alpha = G_{\mathrm{wire}}/G_{\mathrm{contact}}$ is used to describe the mesoscopic effects in the case when the contacts between the ring and the electrodes have a tunnelling character. G_{wire} and G_{contact} are the classical wire and contact conductances. When the parameter α is small, $\alpha \ll 1$, it represents the probability of electron's resting in the ring after each flight through a contact region. An isolated ring is described by $\alpha \gg 1$.

By analogy with [5], we can obtain the relation between the quantum corrections to the conductance and the cooperon in a variable magnetic field

$$\delta G(t) = -\frac{4e^2 D}{\pi\hbar V_0 L}\int \mathrm{d}x \int \mathrm{d}\eta \int \mathrm{d}\epsilon \nabla N(\epsilon,x) C^{\eta-t}_{\eta,-\eta}(x,x). \tag{3}$$

Here V_0 is the voltage drop across the sample and $N(\epsilon, x)$ is the distribution function of electrons in the ring, satisfying the following equation and boundary conditions at the contacts, analogous to those in (2)

$$\begin{aligned}
&\partial_x^2 N(\epsilon, x) = 0 \\
&\pm\alpha L \partial_x N = N - N_F\left(\epsilon + \frac{eV_0}{2}\right) \qquad \text{at} \quad x = +0,\ L-0 \\
&\pm\alpha L \partial_x N = N_F\left(\epsilon - \frac{eV_0}{2}\right) - N \qquad \text{at} \quad x = \frac{L}{2} - 0,\ \frac{L}{2} + 0
\end{aligned} \tag{4}$$

where '+' and '−' correspond to, respectively, 'up' and 'down' semicirclets. $N_F(\epsilon \pm eV_0/2)$ are the Fermi distribution functions for the source and the drain.

Solving (4) we find

$$\partial_x N(\epsilon, x) = \frac{\pm 2eV_0}{L(1+4\alpha)}\delta(\epsilon - \epsilon_F). \tag{5}$$

Substituting (5) into equation (3) and taking into account that in the case of weak tunnelling $\alpha \gg 1$, we obtain†

$$\delta G(t) = -\frac{e^2 D}{\alpha\pi L}\int C^{\eta-t}_{\eta,-\eta}(x, x)\,\mathrm{d}\eta. \tag{6}$$

Taking account of the periodic boundary conditions along the x-axis, the solution of equation (1) in a uniform high-frequency field linearly depending on time ($H = h_0 t$) is given by

$$\begin{aligned}
C^t_{\eta,\eta'}(x, x) = \frac{1}{L}\sum_{m=-\infty}^{\infty}\Theta(\eta - \eta') \\
\times \exp\left\{-\frac{\eta - \eta'}{\tau_\varphi} - D\int_{\eta'}^{\eta}\mathrm{d}\eta_1\left[p_x^m - \frac{e}{c}A_x\left(t - \frac{\eta_1}{2}\right) - \frac{e}{c}A_x\left(t + \frac{\eta_1}{2}\right)\right]^2\right\}
\end{aligned} \tag{7}$$

where $p_x^m = 2\pi m/L$. We use here a gauge with the only component $A_x(t) = \Phi(t)/L$. Substituting equation (7) in (6), we obtain

$$\delta G(t) = -\frac{e^2}{4\alpha\pi^3}\left(\frac{\tau_0}{\tau_R}\right)^{2/3}\sum_m F(x_m, y) \tag{8}$$

$$F(x, y) = \int_0^\infty \exp\left\{-\left[(x - \eta)^2 + y\right]2\eta\right\}\mathrm{d}\eta\,. \tag{9}$$

We use the notation $x_m = (\tau_0/\tau_R)^{1/3}(m + t/\tau_0)$, $y = (\tau_0/\tau_\varphi)(\tau_R/\tau_0)^{1/3}$, $\tau_R = R^2/D$.

The function $F(x, y)$ oscillates with a period τ_0. It reaches a maximun near $x = 0$, and has the following asymptotes:
at $x \to -\infty$

$$F(x, y) \simeq \frac{1}{2(x^2 + y)} \tag{10}$$

at $x \to +\infty$

$$F(x, y) \simeq \sqrt{\frac{\pi}{2x}}\exp(-2xy) + \frac{1}{2(x^2 + y)}. \tag{11}$$

These approximations are valid when $\tau_0 \gg \tau_R$.

† Hereafter we assume $\hbar = 1$. We restore $\hbar$ in the final formulae.

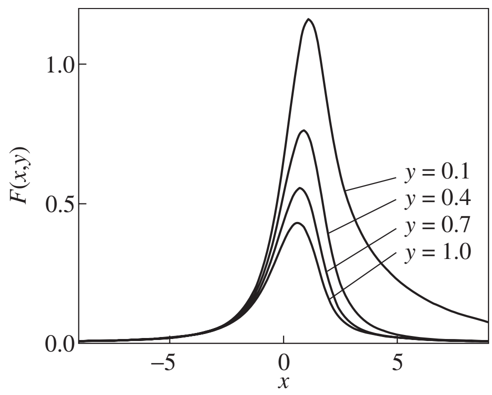

Figure 2. Function $F(x, y)$ at different values of the parameter y.

We can see from (10) and (11) that $F(x, y)$ is an asymmetrical function of time. In particular, when $\tau_0 \gg \tau_\varphi$, the asymptotes coincide and $F(x, y)$ becomes symmetrical. The dependence of $F(x, y)$ on the parameter x at different values of y is depicted in figure 2. This corresponds to the case when $\tau_0 < \tau_\varphi$ and the asymmetry arises from the conservation of the cooperon phase at a time of order of τ_0. The asymmetry disappears in the opposite quasistationary limit, and the quantum corrections coincide with those in stationary case.

Replacing in (8) the summation over m by integration with the use of the Poisson summation formula, we obtain the Fourier representation of the quantum corrections to the conductance

$$\delta G(t) = \delta G_0 + \sum_{k=1}^{\infty}\left[\delta G_k^a \sin\left(\frac{2\pi k t}{\tau_0}\right) + \delta G_k^b \cos\left(\frac{2\pi k t}{\tau_0}\right)\right] \tag{12}$$

where

$$\begin{aligned}\delta G_0 &= -\frac{e^2}{4\alpha\pi^3}\int_0^\infty \sqrt{\frac{\pi}{2\eta\tau_R}}\exp\left(-\frac{2\eta}{\tau_\varphi}\right)\mathrm{d}\eta \\ \delta G_k^{a,b} &= -\frac{e^2}{2\alpha\pi^3}\int_0^\infty \sqrt{\frac{\pi}{2\eta\tau_R}}\exp\left(-\frac{2\eta}{\tau_\varphi} - \frac{\pi^2 k^2 \tau_R}{2\eta}\right)\left\{\begin{array}{c}\sin\,(2\pi k\eta/\tau_0)\\ \cos\,(2\pi k\eta/\tau_0)\end{array}\right\}\mathrm{d}\eta\,.\end{aligned} \tag{13}$$

The superscripts a and b refer to the upper and lower lines in the square brackets in (13), respectively.

Integration of equation (13) yields

$$\begin{aligned}\delta G_0 &= -\frac{e^2}{8\alpha\pi^2}\frac{L_\varphi}{R} \\ \delta G_k^{a,b} &= -\frac{e^2}{4\alpha\pi^2}\left\{2\tau_R\left[\frac{1}{\tau_\varphi^2} + \left(\frac{\pi k}{\tau_0}\right)^2\right]\right\}^{-1/2}\exp\left(-\pi k Z_+\sqrt{2\tau_R}\right) \\ &\quad\times\left(Z_+\left\{\begin{array}{c}\sin\left(\pi k Z_-\sqrt{2\tau_R}\right)\\ \cos\left(\pi k Z_-\sqrt{2\tau_R}\right)\end{array}\right\} \pm Z_-\left\{\begin{array}{c}\cos\left(\pi k Z_-\sqrt{2\tau_R}\right)\\ \sin\left(\pi k Z_-\sqrt{2\tau_R}\right)\end{array}\right\}\right) \\ Z_\pm^2 &= \left[\frac{1}{\tau_\varphi^2} + \left(\frac{\pi k}{\tau_0}\right)^2\right]^{1/2} \pm \frac{1}{\tau_\varphi}\,.\end{aligned} \tag{14}$$

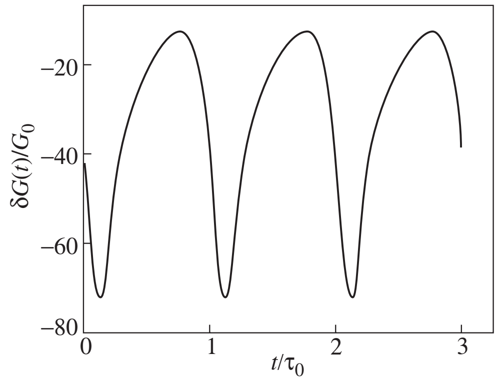

Figure 3. Quantum corrections $\delta G(t)$ (in units of $G_0 = e^2/8\pi^2\alpha$) as a function of time. Here $\tau_\varphi = 10^{-8}$ s, $\tau_0 = 8 \times 10^{-9}$ s, $\tau_R = 10^{-11}$ s.

In the quasistationary case, $\tau_0 \gg \tau_\varphi$, we obtain

$$\delta G_0 = -\frac{e^2}{8\alpha\pi^2}\frac{L_\varphi}{R} \qquad \delta G_k^a = 0 \qquad \delta G_k^b = -\frac{e^2}{4\alpha\pi^2}\frac{L_\varphi}{R}\exp\left(-\frac{2\pi kR}{L\varphi}\right). \tag{15}$$

In the opposite case, with $\tau_0 \ll \tau_\varphi$,

$$\begin{aligned}\delta G_0 &= -\frac{e^2}{8\alpha\pi^2}\frac{L_\varphi}{R}\\ \delta G_k^{a,b} &= -\frac{e^2}{4\alpha\pi^2}\sqrt{\frac{\tau_0}{2\pi k\tau_R}}\exp\left(-\sqrt{\frac{2\tau_R}{\tau_0}}(\pi k)^{3/2}\right)\\ &\quad\times\left[\cos\left(\sqrt{\frac{2\tau_R}{\tau_0}}(\pi k)^{3/2}\right) \pm \sin\left(\sqrt{\frac{2\tau_R}{\tau_0}}(\pi k)^{3/2}\right)\right].\end{aligned} \tag{16}$$

We have strong suppression of the harmonics with $k \neq 0$ in (15) at $R \gg L_\varphi$ and in (16) at $\tau_R \gg \tau_0$.

The time dependence of $\delta G(t)$ in the case $\tau_0 \ll \tau_\varphi$ is depicted in figure 3. Summing (12) over k and using (15), we obtain the well known expression for the quantum corrections in a stationary magnetic field [10]

$$\begin{aligned}\delta G(t) &= -\frac{e^2}{8\alpha\pi^2\hbar}\frac{L_\varphi}{R}\frac{\sinh\left(2\pi R/L_\varphi\right)}{\cosh\left(2\pi R/L_\varphi\right) - \cos\left(2\pi t/\tau_0\right)}\\ &= -\frac{e^2}{8\alpha\pi^2\hbar}\frac{L_\varphi}{R}\frac{\sinh\left(2\pi R/L_\varphi\right)}{\cosh\left(2\pi R/L_\varphi\right) - \cos\left(2\pi Vt/\Phi_0\right)}.\end{aligned} \tag{17}$$

3. Discussion

Let us discuss the conditions under which the Aharonov–Bohm effect is observed in a nonstationary magnetic field. Firstly, the condition $\tau_0 < \tau_\varphi$ must be fulfilled. Hence, the time derivative of the magnetic field must be large enough, so that $\dot{H} > \Phi_0/(d^2\tau_\varphi)$.

In typical strongly doped semiconductors, $\tau_\varphi \sim 10^{-9}$–$10^{-8}$ s (at $T = 1$ K). Then, for a ring of size $d \simeq 10^{-6}$ m, $\dot{H} > 10^5$–10^6 T s^{-1}. This condition is valid, for instance, if

UHF techniques are used to create a nonstationary magnetic field, in particular, by placing the sample under study in a resonator. The maximum fields in evacuated resonators exceed 10^8 V m^{-1}. If the characteristic field in the resonator $H \sim 10^{-3}$ T ($E \sim 3 \times 10^5$ V m^{-1}) and characteristic frequencies $\omega \sim 10^8$–10^9 Hz, we obtain $\dot{H} \sim 10^6$–10^7 T s^{-1}.

However, a strong electric field in the resonator may disturb the measurements. Therefore, a resonator with $E_{\max}$ and $H_{\max}$ having a spatial shift should be used, with the sample located at that place of the resonator where the electric field approaches zero.

In the case of a $\lambda/4$ shift, the electric field variation at the opposite ends of the ring $\Delta E \simeq E_{\max} d/\lambda \simeq (0.5$–$5.0) \times 10^{-6} E_{\max}$. For $E_{\max} \sim 3 \times 10^5$ V m^{-1} $\Delta E \simeq 10^{-1}$–1 V m^{-1} and $\Delta V \simeq 10^{-7}$–10^{-6} V. At the ends of a sample of size $\sim 10^{-4}$ m $\Delta V \simeq 10^{-3}$–10^{-4} V.

The influence of such a voltage drop is expected to be not too essential. An important problem is how can voltage pickups by measuring wires and electric breakdown in measurements be ruled out. Voltage pickups can be reduced by careful fabrication and installation of the electrode system, necessary to compensate for a large possible field in the resonator with an accuracy of 10^{-7}–10^{-6}. We hope that this can be done by the methods of micrometer technology. It is desirable to use a sample consisting of a planar insulating substrate on whose surface both the mesoscopic ring and the conductor electrodes are prepared. The plate should be placed in the resonator with micron accuracy along the plane on which the electric field components are zero. For a parallelepiped resonator and waves of magnetic type ($E_z = 0$), the electric field components E_x and E_y vanish in the middle of the rib perpendicular to the xy-plane, with the magnetic field component H_z being there at a maximum. Therefore, a plate with a ring can be placed close to this point perpendicularly to the z-axis. In this case the measuring leads can be brought out of the resonator through a slot in the resonator wall, which also reduces the stray voltage pickup. Also, it is desirable to ensure compensation of the voltage pickup by the zero spot method.

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